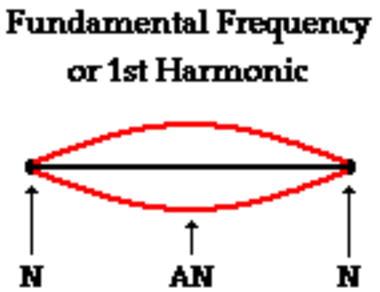
How does the tension of a string affect its fundamental frequency?

Aim: To find out the relationship between tension and the fundamental frequency of the string, and to find out the relationship between the tension and the wave speed.

Hypothesis: As tension increases, the fundamental frequency increases.

Background Information:
When an object or string is vibrating at one of its natural frequencies, or otherwise known as the harmonics, it will produce a standing wave pattern. This standing wave pattern is only produced under the condition given earlier, otherwise, its pattern will be irregular and non-repeating.

The lowest frequency generated by a string is known as the fundamental frequency, which is also called the first harmonic of the string. The diagram below shows the standing wave pattern of the fundamental frequency.



[1]

Standing wave pattern is different from a complete wave, as it does not have any crests or troughs. It is actually merely a vibrational pattern of the wave within the string that contains nodes and antidotes instead. Nodes is produced at the ends of the string where there is no displacement as the string is unable to move at the end. Antinode is placed in between two nodes. The interference of two waves is what causes this standing wave pattern. In the 1st harmonic, there is only one half of a wave within the length of the string. This can conclude that the length of wave for the first harmonic is twice the length of string.

There is actually a pattern between the harmonics and the number of waves shown in the pattern. Increasing an extra harmonic result in an additional node and antinode, and thus an additional half a wave within the string. Concluding from above, a relationship can be seen between the wavelength and the length of the string. In fact, if the number of waves in a string is known, then the equation relating the wavelength of the standing wave pattern to the length of the string can be derived. For example: for the third harmonics, there is $\frac{3}{2}$ waves within the string, and thus to find the wavelength, simply multiply the length of the string by $\frac{2}{3}$.

[1] https://www.physicsclassroom.com/class/sound/Lesson-4/Fundamental-Frequency-and-Harmonics

Mersenne’s Laws are a set of rules describing the oscillations of a stretched string, and it is developed by Marin Mersenne, a French Mathematician and music theorist. It theorizes that length, tension and mass per unit length (linear density) of the string will have a direct impact on the frequency of the string. This practical work is going to test one of the theories of how tension will affect the fundamental frequency of the string or not.

Methodology & Setup of Experiment:

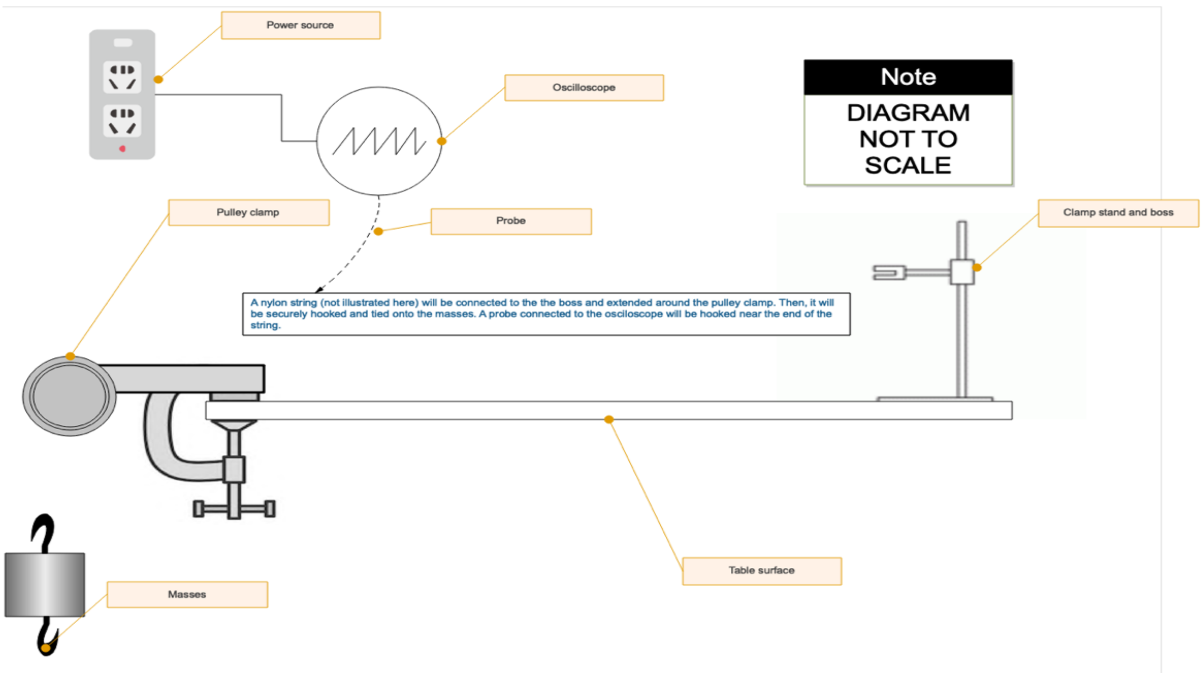
Instruments & Apparatuses used

- Oscilloscope
- Nylon string
- G-clamps (x2)
- Clamp stand & boss
- 100-gram masses (x10)
- Pulley clamp

Setup of Experiment

1. Set up the clamp stand and attach a boss onto the clamp stand.
2. Attach a G-clamp on the clamp stand to secure it in place.
3. Attach the G-clamp onto the table surface at the end of the table.
4. Tie a string around the boss and put it over and across the pulley.
5. Make sure that the string is 0.7 metre in length.
6. Tie the tail part of the string onto the hook that connects the 100-gram masses.
7. Connect the oscilloscope with a nearby power source.
8. Calibrate the oscilloscope so that frequency can be detected and shown on screen.
9. Attach the probe that connect with the oscilloscope towards the end of the string.
10. Make sure that when the string is forced to vibrate (by pressing down on the middle of the string), a frequency can be shown on screen on the oscilloscope.

Diagram



An illustrated diagram that portrays the setup of the experiment in the previous page. It is made using an illustration software. Note that the nylon string is not illustrated in the diagram due to limitations of software.

<u>Methods</u>

1. Start the experiment with one 100-gram mass.
2. Record the lowest and highest frequency recorded on the oscilloscope
3. One by one, add in 9 more 100-gram masses.
4. Repeat the above steps for at least 3 times.

<u>Actual set-up pictures</u>

Figure 1 An overview of the setup of the experiment

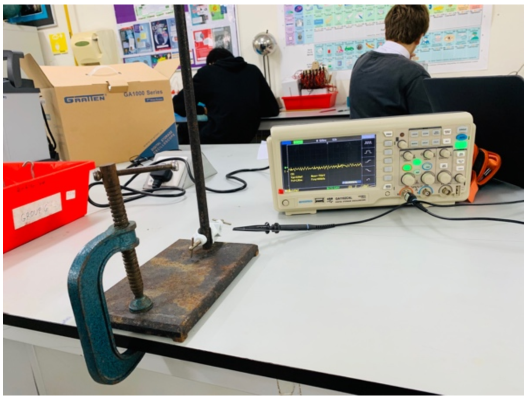


Figure 2 A closer shot on the oscilloscope and clamp stand

Figure 3 A close-up shot on the pulley clamp and the masses

Figure 4 A close-up shot on the probe and the string

Variables:

<u>Constant Variables</u>

These following variables must be kept constant to ensure the fairness and accuracy of the experiment:

- Length of the string – this is to be kept at 0.7m throughout the whole experiment, as length of the string will have a direct impact on the frequency of the string

- Temperature of the string – this is to be kept at 16°C at all times through carrying out the experiment under the same air-conditioned room at constant temperature

- Linear density (mass per unit length) of the string – to be kept at $3x10^{-4}$ kgm^{-1} throughout the whole experiment, by using the same string for all trials.

Independent Variables

- Tension of the string – this is to be changed through adding masses onto the string. Tension can be calculated using this formula: $F = ma$, where F is the tension, m is the mass, and a is the acceleration due to gravity. Therefore, the one that will be changed in the experiment is the masses that will form the tension.

Dependent Variables

- Fundamental frequency – this is to be measured using the oscilloscope to find out the effect of the tension on it. The highest and lowest frequency recorded on the oscilloscope for each mass will be recorded, and then its average will be calculated by adding up the two values together, and then divide the total by two.

Safety Issues & Concerns:

There are several potential safety hazards in my experiment. First of all, which has to be connected to a power source to function, will be used. Therefore, to avoid accidents such as electrocution or fire, the oscilloscope is placed away from all water sources. Besides, the cables connecting the oscilloscope are laid out nicely on the floor, to avoid tripping. There are also few people passing through the place where the experiment is set up to avoid people tripping over the cables or knocking over the apparatus.

In addition to that, some of the apparatuses that are used may be quite heavy such as clamp stand and the oscillator. Therefore, everything is nicely secured on the table or placed further inside the table to avoid them falling and causing physical damages. This especially apply to the clamp stand, which may not be able to support too much amount of force of the masses. In this case, g-clamp has been used to secure and stabilize the stand.

Lastly, nylon string, a crucial object to the experiment, will have sharp edges at the end. So, to avoid inflicting a cut, extra attention and care is put in when handling the string. Also, scissors, which is used to cut the string into lengths, will also inflict injuries, so extra caution is put in when using the scissors. The scissors are also left in somewhere safe such as into the pencil case right after use to avoid hurting other people around.

Raw Data:

Mass / kg (±0.01kg)	Frequency / Hz (±30%)									
	Trial 1			Trial 2			Trial 3			Average Frequency
	Lowest	Highest	Average	Lowest	Highest	Average	Lowest	Highest	Average	
0.10	65.08	67.33	66.21	45.59	56.51	51.05	33.67	39.12	36.40	51.22
0.20	70.48	74.68	72.58	53.49	76.17	64.83	47.89	54.06	50.98	62.80
0.30	77.43	81.63	79.53	72.81	80.12	76.47	59.06	67.10	63.08	73.03
0.40	79.89	88.73	84.31	70.98	78.99	74.99	59.12	75.13	67.13	75.47
0.50	69.44	81.73	75.59	73.89	89.12	81.51	67.34	89.12	78.23	78.44
0.60	78.63	97.63	88.13	73.67	89.10	81.39	68.01	101.60	84.81	84.77
0.70	88.63	109.70	99.17	89.21	102.63	95.92	123.02	127.63	125.33	106.80
0.80	96.97	112.70	104.84	89.16	96.81	92.99	134.03	149.68	141.86	113.23
0.90	112.63	119.34	115.99	119.97	136.20	128.09	129.63	137.65	133.64	125.90
1.00	117.68	122.43	120.06	123.60	125.01	124.31	126.02	142.22	134.12	126.16

Table 1 Table data for data collection

The averages for each trial are found by adding the lowest and highest value together, then divide it by 2. Then, the overall averages are found by adding up the three individual averages from each trial, then divide it by 3.

Data Manipulation:

Using Mersenne's Law formula:

$$f = \frac{\sqrt{\frac{F}{W}}}{2L}$$

where f is the fundamental frequency, F is the force (or tension) in this case. The constants are W, the linear density and L, length of the string.

Rearranging the formula into the form of y=mx+c, and getting rid of the square root by squaring the whole formula, the formula becomes:

$$f^2 = \frac{1}{4L^2W} \times F$$

With the new formula, f^2 is the y-axis, F is the x-axis, and $\frac{1}{4L^2W}$ is the gradient m. The y-intercept c will be 0, which will be the value of f^2 when F = 0. By squaring the formulas, the graph will, therefore, be plotted according to these new criteria for a linear relationship between the axes.

The table below summarises the data of tension (F) and the squared of frequency (f^2):

Tension / N	Percentage Uncertainty of Tension / %	Average Frequency / Hz (±30%)	Squared of frequency / Hz^2 (±60%)
0.981	10.0	51.22	2623.15
1.962	5.00	62.80	3943.21
2.943	3.33	73.03	5332.65
3.924	2.50	75.47	5696.22
4.905	2.00	78.44	6152.83
5.886	1.67	84.77	7186.52
6.867	1.43	106.80	11406.95
7.848	1.25	113.23	12819.90
8.829	1.11	125.90	15851.65
9.810	1.00	126.16	15916.35

Figure 2 Table data showing force and frequency square.

The tension is calculated through the formula:

$$F = ma$$

where F is the force (tension), m is the mass, and a is the acceleration, considered in this case to be 9.81ms^{-2}, the acceleration due to gravity.

The percentage uncertainty of tension is basically the percentage uncertainty of mass, which is calculated through, dividing the absolute uncertainty of the mass (0.01kg)by the masses individually, then multiply by 100 to get the percentage. In this case, the acceleration due to gravity is considered to not have any uncertainty, as it is the universal value for the acceleration due to gravity on the surface of the Earth.

For example:

$$\Delta\%Tension = \Delta\%Mass + \Delta\%Acceleration$$

$$\frac{0.01}{0.1} \times 100 + 0 = 10\%$$

So, the tension when the mass is 0.1kg has a percentage uncertainty of 10%.

The percentage uncertainty of the frequency square is ±60%, which is calculated by multiplying the percentage uncertainty of frequency (30%), given in the user manual of the oscillator, by 2. The calculations are written below:

$$\Delta\% frequency\ sqaured = \Delta\% frequency \times 2$$
$$30\% \times 2 = 60\%$$

Graph:

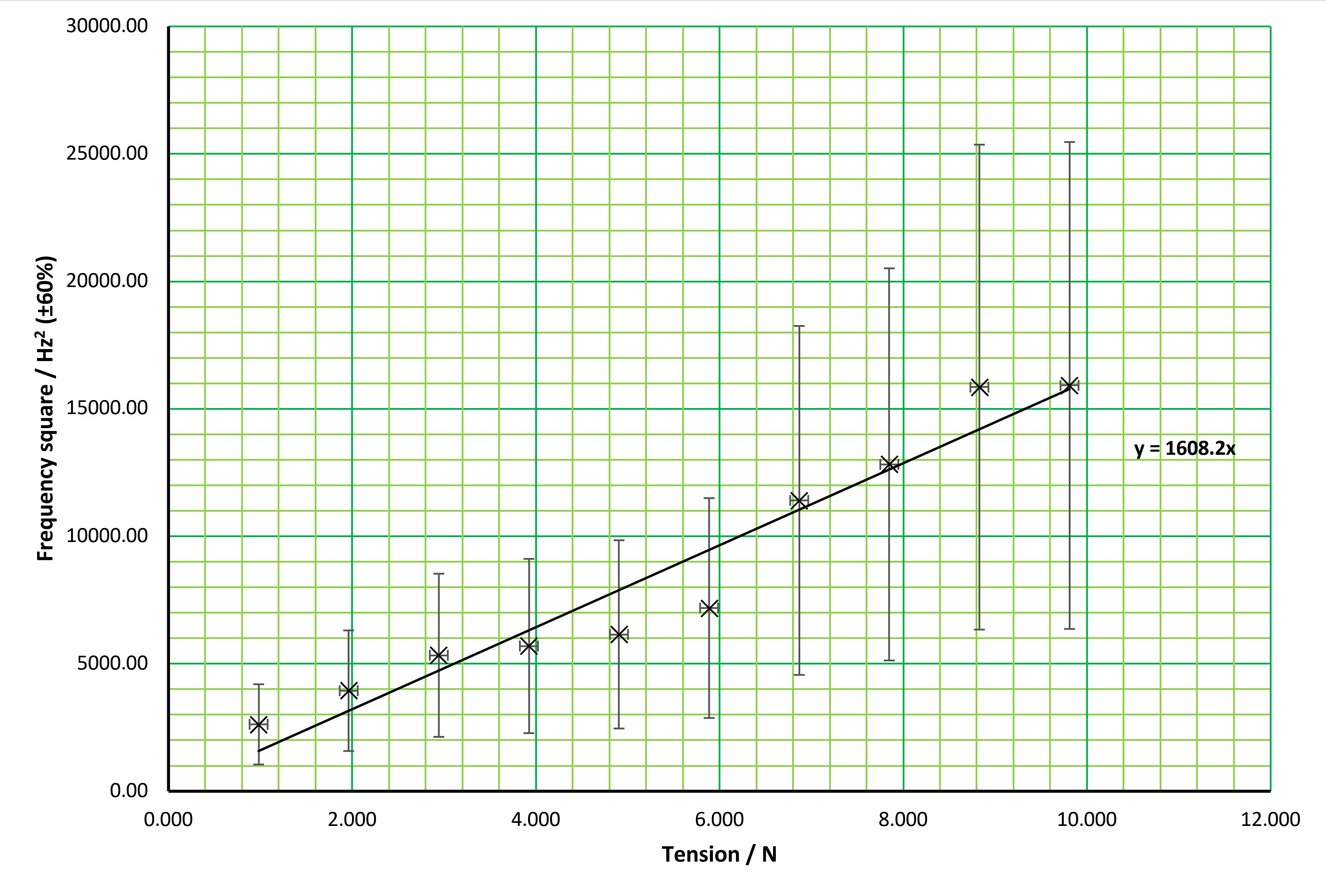


Figure 3 Graph showing relationship between tension and frequency square, with error bars and line of best fit.

The error bars for frequency square on the y-axis is ±60% of the value.

The error bars for tension on the x-axis have different percentage uncertainties, but the same absolute uncertainties for all values, which is ±0.9681 N.

For example when using tension of 6.867, the calculations for the absolute uncertainty is:

$$6.867 \times 1.43\% = 0.0981$$

When the same calculation is done for each value of tension, the answers are always 0.981.

Error bars calculations

Error bars calculations need to be done for frequency square (y-axis) to consider the uncertainties of the gradient. This is done using the following formula:

$$\frac{(max - min)}{2}$$

Maximum value is:
$(15916.35 \ \times \ 60\%) + 15916.35$ = 25466.16

Minimum value is:
2623.15 – $(2623.15 \ \times \ 60\%)$ = 1049.26

Error bars calculation is:

$$\frac{25466.16 - 1049.26}{2} = 12208.45$$

Analysis of graph

The graph shows a positive relationship between tension and frequency squared, as the line of best fit is a positive linear straight line. However, line isn't perfectly linear, as there are a few anomalies in the graph that does not follow the trendline, especially the last four points, which have gone higher than they are expected to be. This may be due to random errors caused by some sources of errors, which will discussed later in the evaluation.

The formula of the gradient is y=1608.2x, which is calculated automatically using a data analysis software. Therefore, the y-intercept is 0, which means that when tension is 0, the frequency squared is also 0, thus the frequency is also 0.

The gradient of the line is 1608.2, a positive gradient. As mentioned earlier, the gradient is $\frac{1}{4L^2W}$, which are the constants of this experiment. Substituting the values of *L* and *W into the formula:*

$$\frac{1}{4(0.7)^2 \times 3 \times 10^{-4}} = 1701$$

The calculated value for the gradient is 1701, which is very close to the actual gradient of 1608.2, after considering the uncertainties of the gradient calculated earlier. The gradient may not be exactly the same due to some sources of errors that cause systematic errors, which will be discussed later as well in the evaluation

Conclusion:

Tension is positively correlated to frequency squared. This means that as tension increases, the fundamental frequency also increases. Thus, the experiment is consistent with the hypothesis, so the hypothesis is proved correct.

Despite there is a significant uncertainties (up to 60%) given by the frequency squared due to the limitations of the oscillator, and that the gradient also has a rather large uncertainty, all the possible trendlines suggest a positive linear relationship, thus suggesting that the hypothesis is correct either way.

All in all, force is directly proportional to fundamental frequency squared, or in other words, fundamental frequency is directly proportional to squared root of force.

Further Application:
This conclusion can be easily applied to real life, especially in string musical instruments such as guitars and harps, where when more tension is applied during tuning and the making of the instruments, people can know that the frequency will be higher, thus raising the pitch of the string.

Evaluation:

Limitations & weaknesses of the practical:

- The oscilloscope used in the practical may not provide accurate reading of the frequency due to the physical limitations of the machine itself, thus may cause the data collected to be unreliable (due to systematic error) and have a large uncertainty. The improvement for this is discussed later on the “Sources of Errors” section.

- Even though a nylon string is used to try to recreate the first harmonics, it is unreliable to judge whether or not the string is in the first harmonics since the only way to judge it is to observe the string and make sure that there is only half a wavelength shown. This means that the practical may not be as accurate since the result may refer to other harmonics besides the first harmonic, causing a systematic error. This can be overcome via using an electric oscillator, which generates a periodic movement automatically with more accuracy.

- The pulley may provide a systematic error to the data collected since the pulley is not entirely frictionless, so that means that the acceleration downwards and the force will be larger than what is expected. This limitations couldn’t be overcome as it is impossible to create a frictionless pulley.

Sources of Errors & Improvements:

Sources of Errors	**Improvements**
The temperature may be difficult to be kept constant at 16 degree at all times, which may affect the precision of the result, as it affects the fundamental frequency of the string since it is difficult to control the slight changes in room temperature.	This source of error can be reduced via carrying out the experiment in a room with controlled air-conditioning so that the temperature can be kept constant at all times to avoid the results being limited by random errors. The room should ideally be isolated since any people / object going near it will affect the temperature.

Length of string is measured using eyes and a ruler, so the measurement may not be exactly 0.7m due to human reaction error, which will result in accuracy (systematic) error in the result.	This can be improved via using a laser measuring device, which provide greater accuracy, reducing the errors.
Linear density of the string may not be exactly $3x10^{-4}$ kgm^{-1} due to the possible error in length of string, as discussed earlier. In addition, it can also occur due to the error in mass of string. The mass of the string is determined using an electronic weighing machine, but it only shows up till 1 decimal places of the result, which may cause errors with things that are very light, such as the strings used in this case. It is because light objects are better if expressed in more decimal places for the accuracy of the experiment.	The errors in measuring the mass of the string can be improved by using a scale or weighing machine of a greater precision, or that provides more decimal places, so that a more accurate linear density of the string can be calculated.
The oscillator may not give consistent result each time due to the physical limitations of the machine itself, thus giving a 30% uncertainty for the frequency recorded. The uncertainty is also very large as compared with the actual values recorded being quite small numbers.	This can be improved by using a better oscillator with lower uncertainty.
Each time the string is pressed down possibly with a slight difference of force due to human precision error as it is impossible to apply same pressure for each trials on the string. This may result in random errors throughout the whole experiment.	This can be improved via using an electric oscillator generator, which generates waves / oscillations automatically without human physical input.
The oscillator used shows a range of different results, and I only take the highest and lowest result to find the average. However, due to human reaction error, the highest and lowest may be missed and the values are shown on screen for just a brief second. This will result in random errors in the experiment, as the results are not consistent all the times.	This can be improved by using an oscillator with higher accuracy, or using a computer to automatically generate the highest and lowest frequency to ensure accuracy of the results.
The oscillator may not be calibrated correctly due to the physical limitations of the machine, and also due to the lack of understanding of how the machine functions. This may result in systematic errors, caused by non-zero precision error.	This can be improved by doing a proper research on how to use the model of oscillator online or using the manual to gain a better understanding of how the oscillator functions.

Work cited:

- Henderson, Tom. "Fundamental Frequency and Harmonics." The Physics Classroom. Accessed February 11, 2019. https://www.physicsclassroom.com/class/sound/Lesson-4/Fundamental-Frequency-and-Harmonics.